Product (or resultant) of two bilinear transformation :→

Consider two bilinear transformations

$T_1(z) = w = \frac{az+b}{cz+d}$ such that $ad-bc \neq 0$ —(1)

and $\zeta = \frac{a_1 w + b_1}{c_1 w + d_1}$ s.t. $a_1 d_1 - b_1 c_1 \neq 0$ —(2)

$= T_2(w)$

~~Putting~~ Then $\zeta = T_2(w)$

$= T_2(T_1(z))$

$= T_2\left(\frac{az+b}{cz+d}\right)$

$= \frac{a_1\left(\frac{az+b}{cz+d}\right)+b_1}{c_1\left(\frac{az+b}{c_1 w+d}\right)+d_1}$

$= \frac{(aa_1+b_1c)z+(b_1d+a_1b)}{(c_1a+d_1c)z+(dd_1+bc_1)} = \frac{Az}{Cz}$

Writing $A = aa_1+b_1c$, $B = b_1d+a_1b$

$C = c_1a+d_1c$, $D = dd_1+bc_1$

We get $\zeta = \frac{AZ+B}{CZ+D}$.

Here $AD-BC = (a_1d_1-b_1c_1)(ad-bc) \neq 0$

∴ This is a bilinear transformation which is called the product or resultant of two bilinear transformation.

सितम्बर 2004						
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			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

Theorem 1. The set of all bilinear transformations forms a non-abelian group for the composition of product of transformation

Proof: Let $G = \{ [redacted] T(z) : T(z) \text{ is a bilinear transformation} \}$ be the set of all bilinear transformation

closure property: Then G is called with respect to product of transformation.

I) Associative law : We know that in the case of mappings $T_1(T_2T_3) = (T_1T_2)T_3$

Hence the associative law holds for all $T_1, T_2, T_3 \in G$

) Existence of identity element : The identity transformation $I(z) = z$ is the identity element

IV) Existence of Inverse: $w = T(z) = \frac{az+b}{cz+d} \in G$

has its inverse $z = T^{-1}(w) = \frac{dw-b}{-cw+a}$

for $T^{-1}T(z) = T^{-1}\left(\frac{az+b}{cz+d}\right) = \frac{d\left(\frac{az+b}{cz+d}\right)-b}{-c\left(\frac{az+b}{cz+d}\right)+d}$

$$= \frac{adz+db-bcz-bd}{-caz-cb+caz+ad}$$

$$= \frac{(ad-bc)z}{(ad-bc)} = z = I(z)$$

Similarly $TT^{-1}(z) = I(z)$

Hence $T^{-1}T(z) = TT^{-1}(z) = I(z)$

Hence the inverse of an element exist

V) We know that in the case of mapping $T_1T_2 \neq T_2T_1$ Hence the commutative law does not hold for $T_1, T_2 \in G$